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Kripke-style semantics for pretabular fuzzy logics: Involutive idempotent fuzzy logics*

Eunsuk Yang

【Abstract】This paper addresses Kripke-style semantics for pretabular fuzzy logics, called involutive idempotent fuzzy logics. As its example, we consider the involutive uninorm mingle logic **IUML**. More exactly, we first recall the fuzzy logic **IUML** and its algebraic semantics. We next introduce algebraic Kripke-style semantics for it and consider its pretabularity in the context of Kripke-style semantics.

【Key words】pretabularity; involutive idempotent fuzzy logics, **IUML,** algebraic semantics; fuzzy logic; Kripke-style semantics.

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1. Introduction

This paper aims to introduce a way to deal with pretabular fuzzy logics in the context of Kripke-style semantics. For this, we first recall some historical facts associated with pretabularity in fuzzy logic. An arbitrary logic L is called *pretabular* in case all of its proper extensions have their own finite characteristic models, even though L itself does not (Dunn & Hardegree (2001)). Dunn (1970) first showed that the logic **RM** (the relevance logic **R** with mingle[1]) is pretabular. After one year, Dunn-Meyer (1971) further showed that the logics **G** (Gödel logic) is pretabular. Yang (2019c; 2020a) recently noted that those systems can be regarded as fuzzy logics and verified that some more fuzzy logics such as **IUML** (Involutive uninorm mingle logic) are pretabular.

These investigations have been all considered using algebraic semantics. Namely, Dunn, Meyer, and Yang all studied the pretabularity of the fuzzy logics using algebraic semantics. Associated with it, one interesting fact is that, after introducing algebraic semantics for fuzzy logics, their corresponding Kripke-style semantics have been provided. For example, Esteva-Godo (2001) first introduced algebraic semantics for **MTL** (Monoidal t-norm logic) and then Montagna-Ono (2002) provided its corresponding Kripke-style semantics. In particular, Yang (2016) introduced Kripke-style semantics for **IUML** after

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1) The more exact denotation of this system is $\mathbf{RM^0}$, a version of RM with no propositional constants, introduced by Yang (2014a).

Metcalfe-Montagna (2007) provided its algebraic semantics.

Then, since the pretabularity of **IUML** is considered in the context of algebraic semantics (Yang (2019c)), the following question arises.

Q: Can we consider the pretabularity of some involutive idempotent fuzzy logics such as **IUML** in the context of Kripke-style semantics?

As its answer, we investigate the pretabularity of **IUML** using Kripke-style semantics. Note that Kripke-style semantics for the fuzzy logics considered above have the same structures as their algebraic semantics. Yang called such Kripke-style semantics *algebraic* Kripke-style semantics (see Yang (2014a; 2020b)). Similarly, we also call such semantics algebraic Kripke-style semantics.

As preliminaries, in Section 2, we recall **IUML** together with its algebraic semantics. We then establish algebraic Kripke-style semantics for **IUML** in Section 3. Note Yang (2016) considered such semantics for uninorm-based logics and **IUML** is its one example. However, the semantics was not focussed on this logic and so one cannot easily understand soundness and completeness results for **IUML** provided by such semantics. Hence, we treat again this semantics for **IUML**. In Section 4 we deal with the pretabularity of **IUML** using this semantics.

2. IUML and its algebraic semantics

The logic system **IUML** is based on a propositional language having a set of formulas *Fm* inductively constituted by a set of countable atomic sentences *VAR*, constants **F**, **f**, and binary connectives $\vee$, $\wedge$, $\rightarrow$, together with the defined connectives and constants: $\sim A := A \rightarrow \mathbf{f}$; $A \& B := \sim(A \rightarrow \sim B)$; $A \leftrightarrow B := (A \rightarrow B) \wedge (B \rightarrow A)$; $\mathbf{t} := \sim\mathbf{f}$; $\mathbf{T} := \sim\mathbf{F}$. Moreover, we define $A_{\mathbf{t}} := A \wedge \mathbf{t}$.[2)]

Definition 2.1 (Metcalfe & Montagna (2007)) **IUML** consists of the axiom schemes and rules below.

$A \rightarrow A$ (SI); $(A \wedge B) \rightarrow A$, $(A \wedge B) \rightarrow B$ ($\wedge$-E); $((A\rightarrow B)\wedge(A\rightarrow C)) \rightarrow (A\rightarrow(B\wedge C))$ ($\wedge$-I); $A \rightarrow (A \vee B)$, $B \rightarrow (A \vee B)$ ($\vee$-I); $((A\rightarrow C)\wedge(B\rightarrow C)) \rightarrow ((A\vee B)\rightarrow C)$ ($\vee$-E); $A \rightarrow \mathbf{T}$ (VE); $\mathbf{F} \rightarrow A$ (EF); $(A \& B) \rightarrow (B \& A)$ (&-C); $(A \& \mathbf{t}) \leftrightarrow A$ (PP); $(A \rightarrow B) \rightarrow ((B \rightarrow C) \rightarrow (A \rightarrow C))$ (SF); $(A \rightarrow (B \rightarrow C)) \leftrightarrow ((A \& B) \rightarrow C)$ (RE); $((A \rightarrow B) \wedge \mathbf{t}) \vee ((B \rightarrow A) \wedge \mathbf{t})$ ($PL_{\mathbf{t}}$); $\sim\sim A \rightarrow A$ (DNE); $(A \& A) \leftrightarrow A$ (ID); $\mathbf{t} \leftrightarrow \mathbf{f}$ (FP); $A \rightarrow B, A \vdash B$ (mp); $A, B \vdash A \wedge B$ (adj).

($PL_{\mathbf{t}}$) is for linearity. Note that substructural logics which are complete over linearly ordered models are called fuzzy logics (see e.g. Cintula (2006)).

A *theory* over **IUML** is a set of formulas closed under deduction rules of **IUML**. A *proof* in a theory T over **IUML** is

2) We may instead define $A \rightarrow B$ as $\sim(A \& \sim B)$.

defined as usual (see e.g. Yang (2019c)). T $\vdash$ A, more exactly T $\vdash_{\mathbf{IUML}}$ A, means that T proves A in **IUML**, i.e., there is an **IUML**-proof of A in T.

We, for convenience, use "$\sim$", "$\to$", "$\vee$", and "$\wedge$" both as propositional connectives and as algebraic operators.

For algebraic semantics, we first define IUML-algebras.

Definition 2.2 Let a * b := ~(a $\to$ ~b), $\sim$a := a $\to$ f, and a_t := a $\wedge$ t. An *IUML algebra* is a structure **A** = (A, $\bot$, $\top$, t, f, $\to$, $\wedge$, $\vee$), where (A, $\wedge$, $\vee$, $\bot$, $\top$) is a bounded lattice with the least and greatest elements $\bot$, $\top$; (A, *, t) forms a commutative monoid; a * b $\leq$ c iff b $\leq$ a $\to$ c (residuation); t $\leq$ $(a \to b)_t \vee (b \to a)$ (prelinearity, pl_t); $\sim\sim$a $\leq$ a (double negation elimination, dne); a = a * a (idempotence, id); t = f (fixed-point, fp).

An IUML-algebra is called *linearly ordered* if for each a, b, a $\leq$ b or b $\leq$ a. Let **A** be an IUML-algebra. An ***A****-evaluation* is a function *e* : *Fm* $\to$ ***A*** satisfying: $e(\bigstar(A_1, \cdots, A_n)) = \bigstar^{A}(e(A_1), \cdots, e(A_n))$, where $\bigstar \in$ {**F**, **T**, **t**, **f**, $\to$, $\wedge$, $\vee$} and $\bigstar^{A} \in \{\bot, \top$, t, f, $\to$, $\wedge$, $\vee\}$. A sentence A is called *valid* in **A** whenever t $\leq$ *e*(A) for all ***A***-evaluation *e* and an ***A***-evaluation *e* is called an ***A****-model* of a theory T whenever t $\leq$ *e*(A) for all A $\in$ T.

Theorem 2.3 (Completeness, Yang (2019a)) Let T be a theory over **IUML** and A a sentence. T $\vdash_{\mathbf{IUML}}$ A iff for all linearly ordered **IUML**-algebras ***A*** and an ***A***-evaluation *e*, if *e* is an ***A***-model of T, then t $\leq$ *e*(A).

An IUML-algebra is *standard* if it has [0,1] as its carrier set. We finally recall the following standard completeness.

Theorem 2.4 (Yang (2019a)) For **IUML**, it holds that: T $\vdash_{\mathbf{IUML}}$ A iff for every standard IUML-algebra and an evaluation *e*, t $\leq$ *e*(A) whenever t $\leq$ *e*(B) for each B $\in$ T.

3. Kripke-style semantics

We introduce algebraic Kripke-style semantics for **IUML**. We first define some frames.

Definition 3.1 (i) (Kripke frames, Yang (2014a)) A structure **K** = (K, $\leq$, t) is called a *Kripke frame* if (K, $\leq$) is a partially ordered set having t $\in$ K. The elements of **K** are called *nodes*.

(ii) ((Commutative) operational Kripke frames) A Kripke frame **K** = (K, $\leq$, t, *) is called an *operational* Kripke frame if (K, t, *) is a monoid with unit. An operational Kripke frame is commutative if * is commutative.

(iii) (Residuated commutative operational Kripke frames) A commutative operational Kripke frame is called *residuated* if the set {c: a * c $\leq$ b} has a supremum, denoted by a $\rightarrow$ b for every a, b in K.

(iv) (Pointed, bounded, linear, complete Kripke frames) A Kripke frame is *linear* if (K, $\leq$) is a linearly ordered set;

pointed if it has some element f ∈ K; *bounded* if it has the least and greatest elements; *complete* if ≤ is a complete order.

(v) ((Involutive) UL frames) A *UL* frame is a pointed, bounded linear residuated commutative operational Kripke frame, where * is left-continuous and conjunctive. Let ~a be a → f. A UL frame is *involutive* if ~~a = a.

(vi) (IUML frames) An *IUML frame* is an involutive UL frame satisfying (id) a * a = a and (fp) t = f.

An *evaluation* over a pointed bounded residuated Kripke frame is a forcing relation ⊩ between nodes and atomic sentences, constants, and sentences satisfying the conditions below: for every atomic sentence p,

(AHC) if a ⊩ p and b ≤ a, then b ⊩ p;
(min) ⊥ ⊩ p,

for the propositional constants **t**, **f**, and **F**,

(t) a ⊩ **t** iff a ≤ t;
(f) a ⊩ **f** iff a ≤ f;
(⊥) a ⊩ **F** iff a = ⊥, and

for any sentences,

(∧) a ⊩ A ∧ B iff a ⊩ A and a ⊩ B;
(∨) a ⊩ A ∨ B iff a ⊩ A or a ⊩ B;

(&) a $\Vdash$ A & B iff there are b, c $\in$ K such that b $\Vdash$ A, c $\Vdash$ B, and a $\leq$ b $*$ c;
($\rightarrow$) a $\Vdash$ A $\rightarrow$ B iff for all b $\in$ K, if b $\Vdash$ A, then a $*$ b $\Vdash$ B.

An evaluation over an IUML frame is an evaluation over a pointed bounded residuated Kripke frame such that (max) for any atomic sentence p, {a : a $\Vdash$ p} has a maximum.

An *IUML model* is a pair (**K**, $\Vdash$), where **K** is an IUML frame and $\Vdash$ is an evaluation over **K**. An IUML model (**K**, $\Vdash$) is called *complete* if **K** is a complete frame.

Let (**K**, $\Vdash$) be an IUML model, **K** a set of nodes, and A a sentence. A is called *true* in (**K**, $\Vdash$) if t $\Vdash$ A, and that A is *valid* in the frame **K** (expressed by **K** $\vDash$ A) if A is true in (**K**, $\Vdash$) for every evaluation $\Vdash$ over **K**.

For soundness and completeness for **IUML**, we first note the following lemmas.

Lemma 3.2 (Yang (2016))

(i) (Hereditary Lemma, HL) Let **K** be an algebraic Kripke frame. For every sentence A and for any nodes a, b $\in$ K, if a $\Vdash$ A and b $\leq$ a, then b $\Vdash$ A.

(ii) Let $\Vdash$ be an evaluation on an IUML frame, and A a sentence. Then the set {a $\in$ K : a $\Vdash$ A} has a maximum.

Lemma 3.3 (Yang (2020b)) t $\Vdash$ A $\rightarrow$ B iff for every a $\in$ K, if a $\Vdash$ A, then a $\Vdash$ B.

Proposition 3.4 (Soundness) If $\vdash_{\mathbf{IUML}}$ A, then A is valid in every IUML frame.

Proof: Interesting cases are the axioms (DNE), (ID), and (FP). For (DNE), see Proposition 3.3 in Yang (2019b).

For (ID), by Lemma 3.3, we need to show that a ⊩ A & A iff a ⊩ A. (⇒) By (RE), we can instead assume that a ⊩ A and show that a ⊩ A → A. Assume that a ⊩ A. Since a = a * a, we obtain that a * a ⊩ A and thus a ⊩ A → A by the condition (→). (⇐) We assume that a ⊩ A and show that a ⊩ A & A. Similarly, since a = a * a and so a ≤ a * a, we have that a ⊩ A & A by the condition (&).

For (FP), by Lemma 3.3, we need to show that a ⊩ **t** iff a ⊩ **f**. (⇒) We suppose that a ⊩ **t** and show that a ⊩ **f**. Using the supposition and the condition (t), we have that a ≤ t. Then, since t = f, we obtain that a ⊩ **f** using the condition (f). (⇐) The proof of this direction is analogous. □

For completeness, we note a connection between algebraic Kripke semantics and algebraic semantics for **IUML**.

Proposition 3.5 (i) The {⊥, ⊤, f, t, ∗, →, ≤} reduct of a (complete) linearly ordered IUML-algebra A is a (complete) IUML frame.

(ii) Let **K** = (K, ⊥, ⊤, f, t, ∗, →, ≤) be an IUML frame. Then the structure **A** = (K, ⊥, ⊤, f, t, max, min, ∗, →, ≤) forms an IUML-algebra.

(iii) Let **K** be the $\{\bot, \top, f, t, *, \to, \leq\}$ reduct of a linearly ordered IUML-algebra $\mathbf{A}$, and let e be an evaluation on $\mathbf{A}$. Let for every atomic sentence p and for every $a \in \mathbf{A}$, $a \Vdash p$ iff $a \leq e(p)$. Then $(\mathbf{K}, \Vdash)$ is an IUML model, and for every sentence A and for every $a \in \mathbf{A}$, we obtain that: $a \Vdash A$ iff $a \leq e(A)$.

(iv) Let $(\mathbf{K}, \Vdash)$ be an IUML model and $\mathbf{A}$ the IUML-algebra being defined as in (ii). We define $e(p) = \max\{a \in K : a \Vdash p\}$ for any atomic sentence p. Then for any sentence A, $e(A) = \max\{a \in K : a \Vdash A\}$.

Proof: The proof is analogous to that of Proposition 3.8 in Yang (2012). □

Theorem 3.6 (Strong completeness)

(i) **IUML** is strongly complete w.r.t. the set of IUML frames.

(ii) **IUML** is strongly complete w.r.t. the class of complete IUML frames.

Proof: (i) and (ii) follow from Proposition 3.5 and Theorem 2.3, and from Proposition 3.5 and Theorem 2.4, respectively. □

4. Pretabularity

Here, using IUML frames in place of IUML-algebras, we show that **IUML** is pretabular.

By *0* and *1*, we express $\bot$ and $\top$, respectively, on [0,1] or on its subset with the least and greatest elements 0, 1. We call IUML frames on such a carrier set *IUML^S frames* and IUML^S frames with an element a such that a = ~a *fixpointed*. More precisely,

Proposition 4.1 Let the carrier set S be [0,1] or its subset with the least and greatest elements 0, 1. A fixpointed *IUML^S frame* is an IUML frame with 1/2 satisfying:
T1. a → b = max(1-a, b) if a ≤ b, and otherwise a → b = min(1-a, b);
T2. ~a = 1 - a.

Henceforth, $IUML^S_{[0,1]}$ *frame* is used in order to denote the IUML^S frame on [0,1] and $IUML^S_{2n-1}$ *frame* is used to denote the IUML^S frame whose carrier set is {0, 1/n+1, ⋯, n/n+1, 1}. Generalizing, by *S frame*, we denote any frame whose elements form a chain with the least, greatest, and fixpointed elements, and whose operations are similarly defined.

Note that 1/2 can be regarded as the fixpointed element in S frames since 1/2 = ~1/2. An extension of a logic L is called *proper* in case its theorems are not exactly the same as L.

Definition 4.2
(i) (Tabularity) A logic L is said to be *tabular* if it has some finite characteristic frame.
(ii) (Pretabularity) A logic L is said to be *pretabular* if (a) it is not tabular and (b) every its proper extension has some finite

characteristic frame.

Following Yang (2019c), we verify that **IUML** is pretabular. Let S-algebras be IUML-algebras with the same carrier sets as S frames. First, as Proposition 3.5 in Section 3, we can show the following.

Proposition 4.3 (i) The {1/2, 1/2, 0, 1, ~, →, ≤} reduct of a (complete) linearly ordered S-algebra $\mathcal{A}$ is a (complete) S frame.

(ii) Let **K** = (K, 1/2, 1/2, 0, 1, ~, →, ≤) be an IUML frame. Then the structure **A** = (K, 1/2, 1/2, 0, 1, max, min, *, ~, →, ≤) is an IUML-algebra, where a * b is defined as ~(a → ~b).

(iii) Let **K** be the {1/2, 1/2, 0, 1, ~, →, ≤} reduct of a linearly ordered IUML-algebra $\mathcal{A}$, and let *e* be an evaluation on $\mathcal{A}$. Let for every atomic sentence p and for every a ∈ $\mathcal{A}$, a ⊩ p iff a ≤ *e*(p). Then (**K**, ⊩) is an S model, and for every sentence A and for every a ∈ $\mathcal{A}$, we obtain that: a ⊩ A iff a ≤ *e*(A).

(iv) Let (**K**, ⊩) be an S model, and let $\mathcal{A}$ be the S-algebra being defined as in (ii). We define *e*(p) = max{a ∈ K : a ⊩ p} for every atomic sentence p. Then for any sentence A, *e*(A) = max{a ∈ K : a ⊩ A}.

Using this proposition and the algebraic results in Yang (2019c), we can show the following.

Proposition 4.4 Let **E** be an extension of **IUML**, **K** be an E frame, and a ∈ **K** satisfy t > a. Then, there is a homomorphism h of **K** onto an S frame which is an E frame satisfying that e > h(a).

Proof: The claim follows from Proposition 3.4 in Yang (2019c) and Proposition 4.3. □

Proposition 4.5 For the logic **IUML**, let $IUML_1^K$, $IUML_2^K$, $IUML_3^K$, ⋯ be a relabeling in order of the sequence of $IUML^S$ frames such that $IUML_1^S$, $IUML_3^S$, $IUML_5^S$, $IUML_7^S$, ⋯, i.e., $IUML_{2n-1}^S$, 1 ≤ n ∈ N. If a sentence A is valid in $IUML^K{}_i$, then it is also valid in $IUML^K{}_j$, for any j, j ≤ i.

Proof: Note that each $IUML_j^S$ is a subframe of $IUML_i^S$. Hence, the claim follows. □

Now, we consider a Lindenbaum-Tarski algebra in the context of frame. For a theory T in **IUML**, we define [A] = {B: T $\vdash_{\mathbf{IUML}}$ A ↔ B} and IUML = {[A] : A ∈ *Fm*}. The *Lindenbaum-Tarski frame* $Lind_T$ w.r.t. **IUML** and T is IUML frame having the domain $IUML_T$, operations $\star^{Lind_T}([A_1], \cdots, [A_n]) = [\star(A_1, \cdots, A_n)]$, where ★ ∈ {→, ~}, and identity t, its negation f, and least and greatest elements are [**t**], [**f**], [**F**], and [**T**], respectively.

Given a propositional system **E** and a set of atomic sentences **A**, let **E**/**A** be that propositional system like **E** except that its sentences contain only the atomic sentences in **A**. Obviously, the following holds.

Proposition 4.6 For an extension **E** of **IUML**, **F**(**E**/**A**) forms an E frame and is characteristic for **E**/**A** because non-theorems are not valid under the canonical evaluation e_c mapping any sentence A to [A], the set of all sentences B such that B ↔ A.

Moreover, we obtain the proposition below, using Propositions 4.4 and 4.6.

Proposition 4.7 For an extension **E** of **IUML**, if a sentence A is not a theorem of **E**, there is some $IUML^S$ frame $IUML_n^S$ satisfying that it is an E frame and A is not valid in it.

Proof: Assume that A is not a theorem of **E**. Proposition 4.6 ensures that A is not valid in the E frame **F**(**E**/**A**), where **A** is the set of atomic sentences occurring in A by the canonical evaluation e_c. Then, [A] is undesignated in **F**(**E**/**A**). Thus, by Proposition 4.4, we have some homomorphism h of **F**(**E**/**A**) onto an $IUML^S$ frame $IUML^S$ such that it is an E frame satisfying h([A]) < *e*. This assures that the composition of h and e_c, h ○ e_c(B) = h([B]), is an evaluation falsifying A in $IUML^S$. Here, an $IUML^S$-subframe, the image h(**F**(**E**/**A**)), is finitely generated because it is the homomorphic image of **F**(**E**/**A**) being generated finitely by the elements [p] such that p ∈ **A**. Hence, by the elements [p], this frame is finitely generated and so every finitely generated $IUML^S$-subframe is finite and isomorphic to some

$IUML_n^S$. Therefore, this frame is isomorphic to some $IUML_n^S$. □

We finally prove the pretabularity of **IUML**.

Theorem 4.8 **IUML** is pretabular.

Proof: We first prove that every proper extension of **IUML** has a finite characteristic frame. For this, assume that $IUML_1^K$, $IUML_2^K$, $IUML_3^K$, ⋯ is the sequence of IUML^S frames defined in Proposition 4.5 and I is the set of indices of those IUML^S frames being E frames such that **E** is the given proper extension of **IUML**.

Let first I contain an infinite number of indices. Proposition 4.5 ensures that I contains all indices. Note that every IUML^S frame $IUML_i^K$ is an IUML frame. By Proposition 4.7 and Theorem 3.6, we have that **E** is identical with **IUML**, a contradiction, since we assume that **E** is a proper extension of **IUML**.

Second, let I contain just a finite number of indices. Similarly, Proposition 4.5 assures that we can construct some index i, where I contains exactly those indices less then or equal to i. Then, by construction, we have $IUML_i^K$ as an E frame. Consider a sentence A, which is not a theorem of **E**. Proposition 4.7 ensures that A is not valid in some E frame $IUML_h^M$ and h $\leq$ i by our choice of i. Moreover, by Proposition 4.5, we can ensure that A is not valid in $IUML_i^K$, the finite characteristic frame.

We then need to show that **IUML** does not have any finite characteristic frame. The proof is analogous to that of Sugihara in Sugihara (1955). Therefore, we can assure that **IUML** is pretabular. □

Remark 4.9 The semi-relevance logic $\mathbf{RM^t}$, a version of **RM** with constants **t**, **f** but without constants **T**, **F**, is the system **IUML** dropping the axioms (VE), (EF) and (FP). This system can be regarded as a non-bounded version of **IUML**. Interestingly. $\mathbf{RM^t}$ is not pretabular since its extension may have (FP).

5. Concluding remark

We considered the pretabularity of the logic **IUML** in the context of algebraic Ktipke-style semantics. More precisely, we provided algebraic Ktipke-style semantics for **IUML** and proved its pretabularity using this semantics.

However, we just gave a remark that $\mathbf{RM^t}$ is not pretabular without any exact proof and did not consider other pretabular fuzzy systems using such semantics. These are problems left in this paper.

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전북대학교 철학과, 비판적사고와논술연구소
Department of Philosophy & Institute of Critical Thinking and Writing, Jeonbuk National University
eunsyang@jbnu.ac.kr

선표 논리를 위한 크립키형 의미론: 누승적 멱등 퍼지 논리

양 은 석

이 논문에서 우리는 누승적 멱등 퍼지 논리라고 불리는 선표 퍼지 논리의 크립키형 의미론을 다룬다. 이의 한 예로 누승적 멱등 퍼지 논리 체계 **IUML**을 검토한다. 보다 구체적으로 우리는 이 체계와 이 체계의 대수적 의미론을 먼저 소개한다. 다음으로 이 체계를 위한 대수적 크립키형 의미론을 제공하고 이의 선표선을 크립키형 의미론의 문맥에서 다룬다.

주요어: 선표성, 누승적 멱등 퍼지 논리, **IUML**, 대수적 의미론, 퍼지 논리, 크립키형 의미론