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Micanorm-Based Logics with Fixed-Point*

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【Abstract】Standard completeness for fixed-pointed mianorm-based logics were recently introduced by Yang. Here, we extend it to logics with commutativity, i.e., fixed-pointed micanorm-based logics. More precisely, some fixed-pointed micanorm-based logics and their algebraic semantics are first discussed. Next, several examples of fixed-pointed micanorms are introduced. Standard completeness results are finally provided for those logics.

【Key Words】Fixed-point, Micanorm, Substructural logic, Fuzzy logic, Standard completeness

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1. Introduction

After Běhounek and Cintula (2006) first called logics being complete on the unit interval [0, 1] core fuzzy logics, many substructural core fuzzy logic systems have been introduced (see e.g. Cintula, Horčík, and Noguera (2013, 2015)). In particular, Yang (2016) introduced **MIAL** (Mianorm logic) and some non-commutative extensions of it as mianorm-based logics and model-theoretically provided completeness on [0, 1], so called standard completeness, for them. Recently, Yang (2020b) extended it to mianorm-based logics with fixed-point.

Note that before considering **MIAL**, Yang (2015) introduced **MICAL** (Micanorm logic) and some axiomatic extensions of it as micanorm-based logics and considered their standard completeness results. Although **MICAL** is obtained from **MIAL** by adding commutativity and thus the class of micanorm-based logics forms a subclass of mianorm-based logics, he did not investigate fixed-pointed micanorm-based logics in Yang (2020b). Note that extensions of **MIAL** do not necessarily form core fuzzy logics. For instance, $\mathbf{MIAL_a}$, the **MIAL** with associativity, is not standard complete (see Horĉík (2011), Yang (2016)). This gives rise to a question as follows.

Q: Can we establish standard completeness for fixed-pointed micanorm-based logics as commutative extensions of those fixed-pointed mianorm-based logics?

This paper gives a positive answer to the question. To complete it, Sect. 2 discusses commutative extensions of the fixed-pointed logics based on mianorms introduced in Yang (2020b) and algebraic semantics for them. Sect. 3 introduces some examples of fixed-pointed micanorms. Sect. 4 establishes standard completeness for those fixed-pointed micanorm-based logics.

2. Fixed-Pointed Micanorm-Based Logics and Their Algebraic Semantics

Let £ be a countable propositional language, which has a set of sentences Fm inductively built on a set of variables VAR, constants F, T, t, f, connectives →, ∨, ∧, &, and the defined notations and connective: P^n := ((⋯(P&P)&⋯&P)&P, n factors; P_t := P ∧ t; and P ↔ Q := (P → Q) ∧ (Q → P). Fixed-pointed micanorm logics are introduced on £.

Definition 2.1 (i) (MICAL, Yang (2015)) The following are axioms and rules for MICAL:

(P ∧ Q) → P, (P ∧ Q) → Q;
((P → Q) ∧ (P → R)) → (P → (Q ∧ R));
P → (P ∨ Q), Q → (P ∨ Q);
((P → R) ∧ (Q → R)) → ((P ∨ Q) → R);
F → P; (t → P) ↔ P; (P_t & Q_t) → (P ∧ Q);
P → (Q → (Q & P)); (P & Q) → (Q & P);
(Q & (P & (P → (Q → R)))) → R;
((P → (P & (P → Q))) & (Q → R)) → (P → R);

$((S\&T)\to(S\&(T\&(P\to Q)_t)))\vee(S'\to(T'\to((T'\&S')\&(Q\to P)_t)))$ (PL);

$P \to Q, P \vdash Q$; $P \vdash P_t$;

$P \vdash (S \& T) \to (S \& (T \& P))$;

$P \vdash S \to (T \to ((T \& S) \& P))$.

(ii) FMICAL is MICAL plus $t \leftrightarrow f$ (fixed-point, *F*).

Definition 2.2 The following are some structural axioms for logics extending FMICAL:

(expansion, *p*) $(P \& P) \to P$

(contraction, *c*) $P \to (P \& P)$

(n-mingle, m_n) $P^n \to P^{n-1}$, $3 \leq n$

(n-contraction, c_n) $P^{n-1} \to P^n$, $3 \leq n$.

For arbitrary $S \subseteq \{p, c, m_n, c_n\}$, $FMICAL_S$ is a non-associative fuzzy logic extending FMICAL.

Expansion *p* is 2-mingle and contraction *c* is 2-contractive.

Definition 2.3 $Ls = \{FMICAL_S: S \subseteq \{p, c, m_n, c_n\}\}$

Remark 2.4 For arbitrary $S \in \{p, c\}$, $MICAL_S$ was introduced in Yang (2015).

Henceforth, the notations "∧," "∨," and "→" are used both as connectives and as operators.

Algebraic structures characterizing $L \in Ls$ can be introduced as follows.

Definition 2.5 (i) (FMICAL-algebra) Let a_t be $a \wedge t$. An

FMICAL-algebra is a structure A = (A, ⊥, ⊤, f, t, ∨, ∧, *, →) satisfying: (A, ⊥, ⊤, ∨, ∧) is a bounded lattice; (A, *, f, t) is a fixed-pointed commutative unital groupoid; (residuation) b ≤ a → c if and only if (iff) a * b ≤ c, for all a, b, c ∈ A; (PL^A) t ≤ ((d*e)→(d*(e*(a→b)$_t$))) ∨ (d'→(e'→((e'*d')*(b→a)$_t$))), for all a, b, d, e, d', e' ∈ A.

(ii) (L-algebras) For any a ∈ A,

• a * a ≤ a (p^A); • a ≤ a * a (c^A);

• $a^n \leq a^{n-1}$, 3 ≤ n (m_n^A); • $a^{n-1} \leq a^n$, 3 ≤ n, (c_n^A).

For any S ⊆ {p^A, c^A, m_n^A, c_n^A}, *FMICAL$_S$-algebras* are defined, along with corresponding inequations. These algebras are all called *L-algebras.*

An L-algebra is called l*inearly ordered* if ≤ is a linear order on A. An *A*-interpretation is defined as a function *i* : *Fm* → *A* such that *i*(#(P_1, ⋯, P_n)) = $\#^A$(*i*(P_1), ⋯, *i*(P_n)), where # ∈ {∨, ∧, →, &, *F, T, t, f*} and $\#^A$ ∈ {∨, ∧, →, *, ⊥, ⊤, t, f}. If t ≤ *i*(P) for all *A*-interpretation *i*, a sentence P is called *valid*. If t ≤ *i*(P) for all P ∈ T, an *A*-interpretation *i* is called an *A-model* of T.

Theorem 2.6 (Strong completeness) Let P be a sentence and T a theory over L ∈ Ls, T $\vdash_L$ P iff for an *A*-interpretation *i* and any linearly ordered L-algebra *A*, t ≤ *i*(P) whenever *i* is an *A*-model of T.

Proof: The claim is a corollary of Theorem 3.1.8 in Cintula and Noguera (2011). □

3. Fixed-Pointed Micanorms and Examples

Henceforth, we use 0, 1, ∂ and e on [0, 1] to denote the elements $\bot$, $\top$, f and t, respectively.

Definition 3.1 (i) (micanorm, Yang (2015)) A map $\bullet$: $[0, 1]^2 \to [0, 1]$ is a *micanorm* if it satisfies the following: for some $e \in [0, 1]$ and for any a, b, c $\in$ [0, 1], (commutativity) a $\bullet$ b = b $\bullet$ a; (monotonicity) a $\leq$ b implies a $\bullet$ c $\leq$ b $\bullet$ c; (identity) $e \bullet a = a \bullet e = a$.

(ii) (Fixed-pointed micanorm) A *fixed-pointed micanorm* (briefly *fmicanorm*) is a micanorm with ∂ satisfying (F^A).

(iii) (L-fmicanorm) Let S $\subseteq$ $\{p^A, c^A, m_n{}^A, c_n{}^A\}$. *S-fmicanorms* are defined, along with corresponding inequations. These fmicanorms are all called *L-fmicanorms.*

A uninorm is an associative micanorm; a uninorm is called a *t-norm* (*t-conorm* resp) if $e = 1$ ($e = 0$ resp); a micanorm is called *conjunctive* if 0 = 0 $\bullet$ 1 = 1 $\bullet$ 0.

Now we introduce some fixed-pointed micanorms. Before introducing examples of L-fmicanorms, notice that a uninorm is expansive, i.e., $a^2 \leq a$, for $a \leq e$ and contractive, i.e., $a \leq a^2$, for $e \leq a$, and has two subalgebras isomorphic to a t-norm and a t-conorm.[1)]

1) Let $\bullet$ be a uninorm and $e \in (0, 1)$. $\bullet : [0, 1]\upharpoonright_{[e,1]}{}^2 \to [0, 1]\upharpoonright_{[e,1]}$ is isomorphic to a t-conorm and $\bullet : [0, 1]\upharpoonright_{[0,e]}{}^2 \to [0, 1]\upharpoonright_{[0,e]}$ to a t-norm (see Yang (2019)).

Let $\bullet$ be a micanorm operator. The first note still works for micanorms since a $\bullet$ a $\leq$ e $\bullet$ a = a if a $\leq$ e and otherwise a = e $\bullet$ a $\leq$ a $\bullet$ a by monotonicity. However, the second one does not necessarily. Here we introduce some simple examples of L-fmicanorms.

Example 3.2 Let e $(= \partial)$ be a fixed-pointed identity.

(i) The operator $\bullet$: [0, 1]^2 → [0, 1] provided by

$$a \bullet b = \begin{cases} \min(1, a+b-e) & \text{if } e \leq a, b; \\ \max(0, a+b-e) & \text{if } a, b \leq e; \\ e & otherwise. \end{cases}$$

is an fmicanorm operator.

(ii) The operator $\bullet_p$: [0, 1]^2 → [0, 1] provided by

$$a \bullet_p b = \begin{cases} \max(0, a+b-e) & \text{if } a, b \leq e; \\ \max(a, b) & \text{if } e \leq a, b; \\ e & otherwise. \end{cases}$$

is an expansive fmicanorm operator.

(iii) The operator $\bullet_c$: [0, 1]^2 → [0, 1] provided by

$$a \bullet_c b = \begin{cases} \min(1, a+b-e) & \text{if } e \leq a, b; \\ \min(a, b) & \text{if } a, b \leq e; \\ e & otherwise. \end{cases}$$

is a contractive fmicanorm operator.

(iv) The operator $\bullet_{m3}$: [0, 1]^2 → [0, 1] provided by

$$a \bullet_{m3} b = \begin{cases} 1 & \text{if } e < a, b; \\ b & \text{if } a = e < b; \\ \min(a, b) & \text{if } a, b \leq e; \\ e & otherwise. \end{cases}$$

is a 3-mingle fmicanorm operator.

(v) The operator $\bullet_{c3}$: $[0, 1]^2 \rightarrow [0, 1]$ provided by

$$a \bullet_{c3} b = \begin{cases} \max(a, b) & \text{if } e \leq a, b; \\ 0 & \text{if } a, b < e; \\ b & \text{if } b < a = e; \\ e & otherwise. \end{cases}$$

is a 3-contractive fmicanorm operator.

Let e = ½ in $\bullet_{m3}$ and consider a = ⅝. Then, $a \neq a^2 = a^3 = 1$. This verifies that $\bullet_{m3}$ is 3-mingle but not 2-mingle, i.e., not expansive, and similarly for $\bullet_{c3}$.

Remark 3.3

(1) The fmicanorm $\bullet$ in (*i*) has the subalgebras isomorphic to the Łukasiewicz t-conorm and t-norm.[2)]

(2) The fmicanorm $\bullet_p$ in (*ii*) has the subalgebras isomorphic to the Łukasiewicz t-conorm and the Gödel t-norm, and the fmicanorm $\bullet_c$ in *(iii)* has the subalgebras isomorphic to the Gödel t-conorm and the Łukasiewicz t-norm.[3)]

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2) Łukasiewicz t-conorm and t-norm are defined as follows: (Łukasiewicz t-conorm) $S^{Ł}(a, b) = \min\{1, a + b\}$; (Łukasiewicz t-norm). $T^{Ł}(a, b) = \max\{0, a + b - 1\}$

3) Gödel t-conorm and t-norm are defined as follows: (Gödel t-conorm) $S^{G}(a, b)$

(3) The fmicanorm $\bullet_{m3}$ in (*iv*) has the subalgebras isomorphic to the drastic sum t-conorm and the Gödel t-norm, and the fmicanorm $\bullet_{c3}$ in (*v*) has the subalgebras isomorphic to the Gödel t-conorm and the drastic product t-norm.[4)]

4. Standard Completeness

Using Yang's semantic construction in Yang (2015), for L $\in$ Ls we establish standard completeness.

Theorem 4.1 (Standard completeness, Yang (2015)) For $\mathrm{MICAL_T}$, T $\subseteq$ {*p, c*}, T $\vdash_{\mathrm{MICAL_T}}$ P iff for all standard $\mathrm{MICAL_T}$-algebras and interpretations *i*, $e \leq i$(P) whenever $e \leq i$(Q) for all Q $\in$ T.

We first show that every linearly ordered finite or countable L-algebra is embeddable into a densely ordered L-algebra.

Proposition 4.2 For any linearly ordered finite or countable L-algebra $\mathcal{A}$ = (A, $\bot$, $\top$, f, t, *, $\rightarrow$, $\vee$, $\wedge$), there exists a countable ordered set B, an operation $\bullet$, and a map i : A $\rightarrow$ B satisfying:

= max{a, b}; (Gödel t-norm) $T^G(a, b)$ = min{a, b}.

4) The drastic sum t-conorm and the drastic product t-norm are defined as follows: (drastic sum t-conorm) $S^D(a, b) = 1$ if $(a, b) \in (0, 1]^2$ and otherwise max(a, b); (drastic product t-norm) $T^D(a, b) = 0$ if $(a, b) \in [0, 1)^2$ and otherwise min(a, b).

(a) B is a densely ordered set having bottom and top elements Bot, Top, and two special elements ∂, e.

(b) (B, $\leq$, $\bullet$, e) forms a linearly ordered monotonic, commutative unital groupoid.

(c) $\bullet$ is conjunctive and left-continuous.

(d) i is an embedding of (A, $\bot$, $\top$, f, t, *, $\vee$, $\wedge$) into (B, Bot, Top, ∂, e, $\bullet$, max, min) such that for all p, q $\in$ A, i(p $\rightarrow$ q) is the residuated pair of i(p) and i(q) in (B, Bot, Top, ∂, e, max, min, $\bullet$).

(e) $\bullet$ satisfies structural properties of *.

Proof: For easiness, we assume that A is a subset of [0, 1] $\cap$ Q having a finite or countable number of elements, together with 0 and 1 as bottom and top elements, respectively. Let B = {(0, 0)} $\cup$ {(p, a): p $\in$ A $\setminus$ {0 (= $\bot$)}, a $\in$ $\mathbf{Q}$ $\cap$ (0, 1]}. Henceforth, the notations $\leq$ and $\leq$ denote the orders in A and B, respectively. For (p, a), (q, b) $\in$ B, define: (p, a) $\leq$ (q, b) iff p < q, or p = q and a $\leq$ b.

For FMICAL$_A$, A $\in$ {c, c_n, 3 $\leq$ n}, define $\bullet$ as follows. For (p, a), (q, b) $\in$ B,

$$(p, a) \bullet (q, b) = \begin{cases} \max((p,a),(q,b)) & \text{if } p*q = p \vee q, p \neq q, \text{ and} \\ & (p,a) \leq e \text{ or } (q,b) \leq e\,; \\ \min((p,a),(q,b)) & \text{if } p*q = p \wedge q, \text{ and} \\ & (p,a) \leq e \text{ or } (q,b) \leq e\,; \\ (p*q, p*q) & otherwise. \end{cases}$$

For FMICAL$_B$, B $\in$ {p, m_n, 3 $\leq$ n}, define $\bullet$ as follows. For (p, a), (q, b) $\in$ B,

$$(p,\ a) \bullet (q,b) = \begin{cases} \max((p,a),(q,b)) & \text{if } p*q = p \vee q, \text{ and } (p,a) \succ e \text{ or } (q,b) \succ e\,; \\ \min((p,a),(q,b)) & \text{if } p*q = p \wedge q, \text{ and } (p,a) \preceq e \text{ or } (q,b) \preceq e\,; \\ (p*q, p*q) & otherwise. \end{cases}$$

For the conditions (a) to (d), see Proposition 2 in Yang (2015). We prove the condition (e). The property fixed-point, i..e., $e = \partial$, follows from the fact that f = t and so ∂ = (f, f) = (t, t) = e. For the structural properties p for FMICAL$_p$ and c for FMICAL$_c$, see Theorem 4 in Yang (2015). For the other conditions for FMICAL$_{cn}$ and FMICAL$_{mn}$, see Proposition 1 in Yang (2019).

Since the other logics are obtained by combinations of the axioms p, c, m_n, c_n, it must be proved that properties corresponding to the combinations of these axioms hold in their corresponding algebras. As an example, FMICAL$_{m4c3}$ is considered here. We have to prove 4-mingle and 3-contractive properties for (p, a) $\in$ X. Since this forms n-potency, i.e., $(p,\ a)^n = (p,\ a)^{n-1}$, for $4 \leq n$, and the n-potency is proved in Yang (2020a), it suffices to show that $(p,\ a)^2 \leq (p,\ a)^3$ and $(p,\ a)^3 = (p,\ a)^4$.

Case 1. (p, a) $\succ$ e. Let $p^2 = p$. Then (p, a) $\preceq$ (p, a) $\bullet$ (p, a) = (p, p) and so $(p,\ a)^2$ = (p, p) = (p, p) $\bullet$ (p, a) = $(p,\ a)^3$. This further implies that $(p,\ a)^3 = (p,\ a)^4$. Let $p^2 \neq p$. Then $t < p < p^2$ and thus (p, a) $\prec$ (p, a) $\bullet$ (p, a) = $(p^2,\ p^2)$, If $p^2 = p^3$, similarly we have that $(p,\ a)^2 = (p,\ a)^3 = (p,\ a)^4$. Otherwise, since $p^2 < p^3 = p^4$, we have that $(p,\ a)^2 \prec (p,\ a)^3$ and $(p,\ a)^3 = (p^2,\ p^2) \bullet (p,\ a) = (p^3,\ p^3) = (p^4,\ p^4) = (p,\ a)^4$.

Case 2. (p, a) $\preceq$ e. Let $p^2 = p$. Then (p, a) = (p, a) $\bullet$ (p,

a) and so $(p, a)^2 = (p, p)^3 = (p, a)^4$. Let $p^2 \neq p$. Then $p^2 < p < t$. Since * is 3-contractive and $p < t$, we have that $p^3 = p^2$ and so $(p, a)^2 = (p, p)^3 = (p, a)^4$. This implies that $(p, a)^2 \leq (p, a)^3$ and $(p, a)^3 = (p, a)^4$. □

Proposition 4.3 For any countable linearly ordered L-algebra, it is embeddable into a standard L-algebra.

Proof: Assume that *A*, *B*, etc. are as in Proposition 4.2 and notice that (B, $\leq$) is a linearly-ordered countable, dense set having bottom and top elements. (B, $\leq$) is order isomorphic to ([**0**, 1] ∩ Q, $\leq$). Let *k* be such an isomorphism and x ● y be $k(k^{-1}(x) \bullet k^{-1}(y))$ for x, y ∈ [0, 1], and $i'(p) = k(i(p))$ for all p ∈ A. If the conditions (a) to (e) in Proposition 4.2 hold, they also hold in [0, 1] ∩ Q, 0, 1, ∂, e, ●, i' in case B, Bot, Top, ∂, e, •, and *i* do. Hence, without loss of generality, we can assume that B = [0, 1] ∩ Q, $\leq$ = $\leq$, and • = ●.

Let x ◆ y be $\sup_{a \in B: a \leq x} \sup_{b \in B: b \leq y} a \bullet b$ for x, y ∈ [0, 1], This definition ensures that ◆ is monotone, conjunctive and has fixed-point and identity. The left-continuity of ◆ can be proved as in Proposition 3 of Yang (2015).

As in Proposition 4.2, for the other structural properties of ◆, we prove the 4-mingle and 3-contractive property as an example. Assume $<x_i : i \in N>$ as an increasing sequence of reals in the unit interval, where $\sup\{x_i : i \in N\} = x$. Then, $x^{n-1} = \sup\{r^{n-1} : r \in [0, 1] \cap Q, r \leq x\}$ and $x^n = \sup\{r^n : r \in [0, 1] \cap Q, r \leq x\}$. For the 4-mingle and 3-contractive property of ◆, we have to show that $x^2 \leq x^3$ and $x^4 = x^3$. Since $r^2 \leq$ r3, it holds that $\sup\{r^2 : r \in [0, 1] \cap Q, r \leq x\} \leq \sup\{r^3 : r \in [0, 1] \cap Q, r \leq x\}$ and so $x^2 \leq x^3$. Moreover, since $x^4 = x^3$, it holds that $\sup\{r^4 : r \in [0, 1] \cap Q, r \leq x\} = \sup\{r^3 : r \in [0, 1] \cap Q, r \leq x\}$ and thus $x^4 = x^3$.

That ◆ extends • is obvious. By (a) to (e), i is an embedding from (A, ⊥, ⊤, t, f, *, ∨, ∧) into ([0, 1], 0, 1, e, ∂, ◆, max, min). We finally notice that it is proved by Proposition 3 in Yang (2015) that ◆ has an implication. □

Theorem 4.4 (Standard completeness) Let L ∈ Ls, T a theory, and P a sentence. T $\vdash_L$ P iff for any standard L-algebra and interpretation *i,* $e \leq i$(P) whenever $e \leq i$(Q) for all Q ∈ T.

Proof: The left-to-right direction is straightforward. For the right-to-left direction, assume P as a sentence satisfying that T $\nvdash_L$ P, A as a linearly ordered L-algebra, and i as an interpretation in A satisfying that t ≤ i(Q) for all Q ∈ T and t > i(P). Let i' be the embedding from A into the standard L-algebra introduced in the proof of Proposition 4.3. We have that i' • i is an interpretation into the standard L-algebra, where $e \leq i'$ • i(Q) but $e > i'$ • i(P). □

5. Concluding Remarks

We studied fixed-pointed micanorm-based logics. We introduced some examples and then considered standard completeness for those logics.

Note that Yang (2021) investigated involutive extensions of fixed-point mianorm-based logics. However, we have not yet considered such extensions. To consider standard completeness for involutive extensions of fixed-point micanorm-based logics remains an open problem.

Note also that this work does not cover core fuzzy logics with knotted axioms $\phi^k \to \phi^n$, for k, n > 1 (see Baldi (2014), Baldi and Ciabattoni (2015)). Standard completeness for such logics also has to be studied.

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고정점을 갖는 미카놈 논리

양 은 석

고정점을 갖는 미카놈 논리를 위한 표준 완전성이 최근 소개되었다. 이 논문에서 우리는 그러한 완전성을 교환 범칙을 갖는 논리들 즉 고정점을 갖는 미카놈 논리들로 확장한다. 보다 구체적으로 고정점을 갖는 미카놈 논리 체계들과 이러한 체계들의 대수적 의미론이 먼저 논의된다. 그리고 그러한 미카놈의 예들을 소개한 후, 마지막으로 고정점을 갖는 미카놈 체계들을 위한 표준 완전성을 제공한다.

주요어: 고정점, 미카놈, 준구조 논리, 퍼지 논리, 표준 완전성