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Algebraic Routley-Meyer-style semantics for the involutive monoidal t-norm logic IMTL*

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【Abstract】 Routley-Meyer-style semantics, called *algebraic* Routley-Meyer-style semantics, was investigated for the fuzzy logic system **MTL**. This paper extends this investigation to involutive systems. More precisely, as an example, we consider such semantics for the fuzzy logic system **IMTL**. First, we recall the involutive monoidal t-norm logic **IMTL** and its algebraic semantics. We next introduce algebraic Routley-Meyer-style semantics for it and then connect this semantics with algebraic semantics.

【Key Words】 (Algebraic) Routley-Meyer-style semantics, Kripke-style semantics, Algebraic semantics, Fuzzy logic, Substructural logic, Involutive logic.

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1. Introduction

As Yang mentioned in his (2019), fuzzy logic deals with *vagueness* and it has wide and narrow meanings made by Zadeh. Namely, “fuzzy logic in its wide sense is fuzzily synonymous with the fuzzy set theory which is the theory of classes with unsharp boundaries and fuzzy logic in its narrow sense is a logical system which aims at a formalization of approximating reasoning.”

One of important trends in semantic investigation of fuzzy logic in the narrow sense is to consider involutive extensions. For instance, after Jenei and Montagna (2002) introduced standard algebraic semantics for the fuzzy logic system **MTL** (Monoidal t-norm logic), Ciabattoni et al. (2002) investigated such semantics for its involutive extension **IMTL** (Involutive **MTL**); Wang (2012) introduced standard algebraic semantics for the system **CnUL** (Uninorm logic with *n*-potency) and then he (2013) extended such semantics to **CnIUL** (Involutive uninorm logic with *n*-potency).

Recently Yang (2018) introduced Routley-Meyer-style semantics equivalent to algebraic semantics for the fuzzy logic system **MTL** (Monoidal t-norm logic). He called such semantics *algebraic* Routley-Meyer-style semantics. Then, a natural question arises as follows:

Q: Can we extend such semantics to its involutive extension?

As its positive answer, we introduce algebraic

Routley-Meyer-style semantics for the involutive monoidal t-norm (based) logic **IMTL**. For this, first, in Section 2, we recall the fuzzy logic **IMTL** and its algebraic semantics. In Section 3, we introduce algebraic Routley-Meyer-style semantics for it and connect this semantics with algebraic semantics. Note that here we in particular introduce an interesting evaluation clause for negation to provide this semantics for **IMTL** (see the condition (¬) in Section 3).

For convenience, we adopt the notations and terminology similar to those in Montagna & Sacchetti (2003; 2004) Yang (2016b; 2017b; 2018), and assume reader familiarity with them (together with the results found therein).

2. Preliminaries: IMTL and its algebraic semantics

In this section, as preliminaries we discuss the system **IMTL** and its algebraic semantics introduced in Yang (2016b; 2017b). The fuzzy logic system **IMTL** is based on a countable propositional language with formulas *Fm* built inductively from a set of propositional variables *VAR*, binary connectives →, &, ∧, ∨, and constants **T**, **F**, with definable connectives:

df1. $\neg\phi := \phi \rightarrow \mathbf{F}$,
df2. $\phi \leftrightarrow \psi := (\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi)$.

For convenience, we henceforth use the customary notations and terminology, and the axiom system to provide a consequence relation.

We first introduce the following axiom schemes and rules for the involutive monoidal t-norm logic **IMTL**.

Definition 2.1 (Yang (2016b)) **IMTL** consists of the following axiom schemes and rules:

A1. $\phi \to \phi$ (self-implication, SI)

A2. $(\phi \wedge \psi) \to \phi$, $(\phi \wedge \psi) \to \psi$ ($\wedge$-elimination, $\wedge$-E)

A3. $((\phi \to \psi) \wedge (\phi \to \chi)) \to (\phi \to (\psi \wedge \chi))$ ($\wedge$-introduction, $\wedge$-I)

A4. $\phi \to (\phi \vee \psi)$, $\psi \to (\phi \vee \psi)$ ($\vee$-introduction, $\vee$-I)

A5. $((\phi \to \chi) \wedge (\psi \to \chi)) \to ((\phi \vee \psi) \to \chi)$ ($\vee$-elimination, $\vee$-E)

A6. $\mathbf{F} \to \phi$ (ex falso quodlibet, EF)

A7. $\phi \to \mathbf{T}$ (verum ex quodlibet, VE)

A8. $\phi \to (\psi \to \chi) \leftrightarrow \psi \to (\phi \to \chi)$ (permutation, PM)

A9. $(\phi \to (\psi \to \chi)) \leftrightarrow ((\phi \,\&\, \psi) \to \chi)$ (residuation, RES)

A10. $(\phi \to \psi) \to ((\psi \to \chi) \to (\phi \to \chi))$ (suffixing, SF)

A11. $(\phi \,\&\, \psi) \to \phi$ (weakening, W)

A12. $(\phi \to \psi) \vee (\psi \to \phi)$ (prelinearity, PL)

A13. $\neg\neg\phi \leftrightarrow \phi$ (double negation)

$\phi \to \psi, \phi \vdash \psi$ (modus ponenes, mp)

$\phi, \psi \vdash \phi \wedge \psi$ (adjunction, adj).

A *theory* over **IMTL** is a set T of formulas closed under a consequence relation. A *proof* in a sequence of formulas whose each member is either an axiom of **IMTL** or a member of T or follows from some preceding members of the sequence using rules of **IMTL**. T $\vdash \phi$, more exactly T $\vdash_{\mathbf{IMTL}} \phi$, means that ϕ

is *provable* in T w.r.t. **IMTL**, i.e., there is an **IMTL**-proof of ϕ in T. If T is empty, denoted by $\vdash_{\mathbf{IMTL}} \phi$, ϕ is said to be a *theorem* in **IMTL**.

Let ϕ^n be $(\cdots((\phi \ \& \ \phi) \ \& \ \phi) \ \cdots \ ) \ \& \ \phi)$, *n* factors. The deduction theorem for **IMTL** is as follows:

Proposition 2.3 (Hájek (1998)) Let T be a theory and ϕ, ψ be formulas.

$T \cup \{\phi\} \vdash_{\mathbf{IMTL}} \psi$ iff there is n such that $T \vdash_{\mathbf{IMTL}} \phi^n \rightarrow \psi$.

For convenience, we use "$\neg$", "$\wedge$", "$\vee$", and "$\rightarrow$" as propositional connectives and as algebraic operators, but context should clarify their meaning.

Varieties of involutive residuated lattice-ordered monoids (briefly, involutive residuated monoids) in the sense of Galatos et al. (2007) provide suitable algebraic structures for **IMTL**.

Definition 2.4 (i) An *integral commutative residuated monoid* is a structure $\mathbf{A} = (A, \top, \bot, \wedge, \vee, *, \rightarrow)$ such that:

(Ⅰ) $(A, \top, \bot, \wedge, \vee)$ is a bounded lattice with top element $\top$ and bottom element $\bot$.

(Ⅱ) $(A, *, \top)$ is a commutative monoid.

(Ⅲ) $y \leq x \rightarrow z$ iff $x * y \leq z$, for all $x, y, z \in A$ (residuation).

(ii) An *MTL-algebra* is an integral commutative residuated monoid satisfying: for all $x, y \in A$,

(PL^A) $(x \rightarrow y) \vee (y \rightarrow x) = \top$.

(iii) An *IMTL-algebra* is an MTL-algebra satisfying: for all $x \in A$,

(DN^A) $\neg\neg x = x$.

By x^n, we denote $x * \cdots * x$, n factors. We can define $\top$ as $\bot \rightarrow \bot$ and $\neg$ as in (df1) using $\rightarrow$ and $\bot$.

As usual, an IMTL-algebra is said to be *linearly ordered* if the ordering of its algebra is linear, i.e., $x \leq y$ or $y \leq x$ (equivalently, $x \wedge y = x$ or $x \wedge y = y$) for each pair x, y.

Definition 2.6 (Evaluation) Let $\mathcal{A}$ be an IMTL-algebra. An *$\mathcal{A}$-evaluation* is a function v : FOR $\rightarrow \mathcal{A}$ satisfying: $v(\phi \wedge \psi) = v(\phi) \wedge v(\psi)$, $v(\phi \vee \psi) = v(\phi) \vee v(\psi)$, $v(\phi \rightarrow \psi) = v(\phi) \rightarrow v(\psi)$, $v(\phi \,\&\, \psi) = v(\phi) * v(\psi)$, $v(\mathbf{F}) = \bot$, $v(\mathbf{T}) = \top$, (and hence $v(\neg\phi) = \neg v(\phi)$).[1)]

Definition 2.7 Let $\mathcal{A}$ be an IMTL-algebra, T be a theory, ϕ be a formula, and K be a class of IMTL-algebras.

(i) (Tautology) ϕ is a *tautology* in $\mathcal{A}$, briefly an *$\mathcal{A}$-tautology* (or *$\mathcal{A}$-valid*), if $v(\phi) = \top$ for each $\mathcal{A}$-evaluation v.

(ii) (Model) An $\mathcal{A}$-evaluation v is an *$\mathcal{A}$-model* of T if $v(\phi) = \top$ for each $\phi \in T$. By *Mod(T, $\mathcal{A}$)*, the class of $\mathcal{A}$-models of T is denoted.

(iii) (Semantic consequence) ϕ is a *semantic consequence* of T w.r.t. K, denoting by $T \vDash_{\mathsf{K}} \phi$, if Mod(T, $\mathcal{A}$) = Mod($T \cup \{\phi\}$,

1) We would like to note that the connective & and constants **T**, **F** correspond to the algebraic operator * and top and bottom elements $\top$, $\bot$.

$\mathcal{A}$) for each $\mathcal{A} \in$ K.

Definition 2.8 (**IMTL**-algebra) Let $\mathcal{A}$, T, and φ be as in Definition 2.7. $\mathcal{A}$ is an ***IMTL**-algebra* iff, whenever φ is IMTL-provable in T (i.e. T $\vdash_{\mathbf{IMTL}}$ φ), it is a semantic consequence of T w.r.t. the set {$\mathcal{A}$} (i.e. T $\vDash_{\{A\}}$ φ), $\mathcal{A}$ a corresponding IMTL-algebra). By $MOD^{l}(IMTL)$, the class of linearly ordered **IMTL**-algebras is denoted. Finally, we write T $\vDash^{1}_{\mathbf{IMTL}}$ φ in place of T $\vDash_{\mathrm{MOD}^{1}(\mathbf{IMTL})}$ φ.

Theorem 2.9 (Strong completeness, Ciabattoni et al. (2002)) Let T be a theory, and φ a formula. T $\vdash_{\mathbf{IMTL}}$ φ iff T $\vDash_{\mathbf{IMTL}}$ φ iff T $\vDash^{1}_{\mathbf{IMTL}}$ φ.

Definition 2.10 An IMTL-algebra is *standard* iff its lattice reduct is [0, 1].

Theorem 2.11 (Strong standard completeness, Ciabattoni et al. (2002)) For **IMTL**, the following are equivalent:

(1) T $\vdash_{\mathbf{IMTL}}$ φ.

(2) For every standard **IMTL**-algebra and evaluation v, if v(ψ) = 1 for all ψ ∈ T, then v(φ) = 1.

3. Algebraic Routley-Meyer-style semantics

3.1 Semantics

We first introduce several Routley-Meyer-style frames.

Definition 3.1 (i) (Algebraic Kripke frame, Yang(2018)) An *algebraic Kripke frame* is a structure **X** = (X, ⊤, ⊥, ≤, ∗) such that (X, ⊤, ⊥, ≤, ∗) is a linearly ordered integral residuated monoid. The elements of **X** are called *nodes*.

(ii) (Algebraic Routley-Meyer frame, Yang(2018)) An *algebraic Routley-Meyer frame* is a structure **X** = (X, ⊤, ⊥, ≤, ∗, R) such that (X, ⊤, ⊥, ≤, ∗) is an algebraic Kripke frame and R ($\subseteq X^3$) satisfies the following postulates: for all a ∈ X,

p1. R⊤⊤⊤[2)]

p2. R⊤aa

p3. Ra⊤a.

(iii) (MTL frame, Yang(2018)) An *MTL frame* is an algebraic Routley-Meyer frame, where ∗ is conjunctive (i.e., ⊥ ∗ ⊤ = ⊥) and left-continuous (i.e., whenever $\sup\{x_i : i \in I\}$ exists, $x * \sup\{x_i : i \in I\} = \sup\{x * x_i : i \in I\}$), and so its residuum → is defined as $x \to y := \sup\{z: x * z \leq y\}$ for all x, y ∈ X, and the following definitions and postulate hold: for all a, b, c, d ∈ X,

df3. R^2abcd := (∃x)(Rabx ∧ Rxcd)

df4. $R^2a(bc)d$:= (∃x)(Raxd ∧ Rbcx)

p_l. R⊤ab or R⊤ba

p_i. Rabc implies R⊤bc.

p_e. Rabc implies Rbac.

p_a. R^2abcd iff $R^2a(bc)d$.

(iv) (IMTL frame) An *IMTL frame* is an MTL-frame satisfying

2) This postulate is redundant because it is a special case of p2 and p3. But for convenience, we take it as a postulate.

(DN^A).

As mentioned in Yang (2018), Definition 3.1 (iii) ensures that an MTL frame has a supremum w.r.t. $*$, i.e., for every x, y $\in$ X, the set $\{z: x * z \leq y\}$ has the supremum. This also ensures that an IMTL frame has a supremum. **X** is called *complete* if $\leq$ is a complete order.

An *evaluation* or *forcing* on an algebraic Routley-Meyer frame is a relation $\Vdash$ between nodes and propositional variables, and arbitrary formulas subject to the conditions below: for every propositional variable p,

(AHC) if $x \Vdash p$ and $y \leq x$, then $y \Vdash p$;
(min) $\bot \Vdash p$;

for the proposition constant **F**,

($\bot$) $x \Vdash \mathbf{F}$ iff $x = \bot$; and

for arbitrary formulas,

($\neg$) $x \Vdash \neg\phi$ iff $\neg x \nVdash \phi$;[3]
($\wedge$) $x \Vdash \phi \wedge \psi$ iff $x \Vdash \phi$ and $x \Vdash \psi$;
($\vee$) $x \Vdash \phi \vee \psi$ iff $x \Vdash \phi$ or $x \Vdash \psi$;
(&) $x \Vdash \phi \,\&\, \psi$ iff there are y, z $\in$ X such that Rzyx, y

[3] Note that, since the basic structure for any IMTL frame has an involutive negation $\neg$, we can introduce the evaluation for negation with "$\neg$x".

$\Vdash \phi$, and $z \Vdash \psi$;

($\rightarrow$) $x \Vdash \phi \rightarrow \psi$ iff for all $y, z \in X$, if Ryxz and $y \Vdash \phi$, then $z \Vdash \psi$.

An evaluation on an IMTL frame is an evaluation further satisfying that (max) for every atomic sentence p, $\{x : x \Vdash p\}$ has a maximum.

Definition 3.2 (i) (Algebraic Routley-Meyer model) An *algebraic Routley-Meyer model* is a pair (**X**, $\Vdash$), where **X** is an algebraic Routley-Meyer frame and $\Vdash$ is a forcing on **X**.

(ii) (IMTL model) An *IMTL model* is a pair (**X**, $\Vdash$), where **X** is an IMTL frame and $\Vdash$ is a forcing on **X**. An IMTL model (**X**, $\Vdash$) is said to be *complete* if **X** is a complete frame and $\Vdash$ is a forcing on **X**.

Definition 3.3 Given an algebraic Routley-Meyer model (**X**, $\Vdash$), a node x of **X** and a formula ϕ, we say that *x forces ϕ* to express $x \Vdash \phi$. We say that ϕ is *true* in (**X**, $\Vdash$) if $\top \Vdash \phi$, and that ϕ is *valid* in the frame **X** (expressed by $\mathbf{X} \models \phi$) if ϕ is true in (**X**, $\Vdash$) for every forcing $\Vdash$ on **X**.

Definition 3.4 An IMTL frame **X** is an **IMTL** frame if all axioms of **IMTL** are valid in **X**. We say that an algebraic Routley-Meyer model **X** is an ***IMTL** model* if **X** is an **IMTL** frame.

3.2 Soundness and completeness

For soundness and completeness for **IMTL**, we first define R as follows:

(df5) Rabc := $c \leq b * a$.

It is easy to show the following lemma.

Lemma 3.5 (Cf, Yang (2016b)) (i) (Hereditary Lemma, HL) Let **X** be an algebraic Routley-Meyer frame. For any sentence ϕ and for all nodes x, y $\in$ **X**, if $x \Vdash \phi$ and $y \leq x$, then $y \Vdash \phi$.

(ii) Let $\Vdash$ be a forcing on an **IMTL** frame, and ϕ a sentence. Then the set $\{x \in X : x \Vdash \phi\}$ has a maximum.

We prove one additional lemma.

Lemma 3.6 $\top \Vdash \phi \rightarrow \psi$ iff for all $x \in X$, if $x \Vdash \phi$, then $x \Vdash \psi$.

Proof: The condition ($\rightarrow$) and the postulate (p3) ensure the left-to-right direction. For the right-to-left direction, we assume that $Rx\top y$ and $x \Vdash \phi$ and show that $y \Vdash \psi$. Then, from the suppositions, we have that $x \Vdash \psi$. Since using (df5) and $Rx\top y$, we have that $y \leq \top * x = x$, we further obtain that $y \Vdash \psi$ by Lemma 3.5 (i). □

Proposition 3.7 (Soundness) If $\vdash_{\mathbf{IMTL}} \phi$, then ϕ is valid in every IMTL frame.

Proof: We prove the validity of (DN) as an example.

(DN) We first show that $\top \Vdash \neg\neg\phi \rightarrow \phi$. By Lemma 3.6, it suffices to assume that $x \Vdash \neg\neg\phi$ and show that $x \Vdash \phi$. Using the condition ($\neg$), we have that $x \Vdash \neg\neg\phi$ iff $\neg x \nVdash \neg\phi$ iff $\neg\neg x \Vdash \phi$; therefore, $x \Vdash \phi$ by (DN^A). Similarly, we can prove the other direction.

The proof for the other cases is left to the interested reader. □

The following proposition shows an important result between postulates for IMTL frames and algebraic (in)equations corresponding to the structural axioms of **IMTL**.

Proposition 3.8 The postulates for IMTL frames introduced in Definition 3.1 are reducible to algebraic (in)equations corresponding to the structural axioms of **IMTL** introduced in Definition 2.1 (see Definition 2.4).

Proof: We need to consider (DN^A). This is immediate since (DN^A) is itself the algebraic equation corresponding to the double negation axiom for **IMTL**.

For the proof for the other cases, see Proposition 3.8 in Yang (2018). □

By a *chain*, we mean a linearly ordered algebra. Note that the relation R can be defined as in (df5) and the postulates for IMTL frames introduced in Definition 3.1 are reducible to their corresponding algebraic (in)equations (see Proposition 3.8). The next proposition connects algebraic Routley-Meyer semantics and algebraic semantics for **IMTL**.

Proposition 3.9 (i) The $\{\top, \bot, \leq, *\}$ reduct of an IMTL chain $\mathbf{A}$ is an IMTL frame, which is complete iff $\mathbf{A}$ is complete.
(ii) Let $\mathbf{X} = (X, \top, \bot, \leq, *)$ be an IMTL frame. Then the structure $\mathbf{A} = (X, \top, \bot, \max, \min, *, \rightarrow)$ is an IMTL-algebra (where *max* and *min* are meant w.r.t. $\leq$).
(iii) Let $\mathbf{X}$ be the $\{\top, \bot, \leq, *\}$ reduct of an IMTL chain $\mathbf{A}$, and let v be an evaluation in $\mathbf{A}$. Let for every atomic formula p and for every $x \in \mathbf{A}$, $x \Vdash p$ iff $x \leq v(p)$. Then $(\mathbf{X}, \Vdash)$ is an IMTL model, and for every formula ϕ and for every $x \in \mathbf{A}$, we obtain that: $x \Vdash \phi$ iff $x \leq v(\phi)$.
(iv) Let $(\mathbf{X}, \Vdash)$ be an IMTL model, and let $\mathbf{A}$ be the IMTL-algebra defined as in (ii). Define for every atomic formula p, $v(p) = \max\{x \in X : x \Vdash p\}$. Then for every formula ϕ, $v(\phi) = \max\{x \in X : x \Vdash \phi\}$.

Proof: The proof is almost the same as Proposition 3.9 in Yang (2018). □

Theorem 3.10 (Strong completeness)
(i) **IMTL** is strongly complete w.r.t. the class of all IMTL

frames.

(ii) **IMTL** is strongly complete w.r.t. the class of complete IMTL frames.

Proof: For (i), Let T be a theory and ϕ be a formula such that T $\nvdash_{\mathbf{IMTL}}$ ϕ. By Theorem 2.9, there is an **IMTL**-algebra **A** and an evaluation v on **A** such that for all $\psi \in$ T, $v(\psi) = \top$ but $v(\phi) < \top$. Take this v as an evaluation for an **IMTL** frame since this frame is the reduct of an **IMTL**-algebra. Then, by Proposition 3.9, we have that for all $\psi \in$ T, $v(\psi) = \top$ but $v(\phi) < \top$. Hence T $\nvDash_{\mathbf{IMTL}}$ ϕ.

For (ii), Let T be a theory and ϕ be a formula such that T $\nvdash_{\mathbf{IMTL}}$ ϕ. By Theorem 2.11, there is a complete **IMTL**-algebra **A** and an evaluation v on **A** such that for all $\psi \in$ T, $v(\psi) = \top$ but $v(\phi) < \top$. Take this v as an evaluation for a complete **IMTL** frame since this frame is the reduct of a complete **IMTL**-algebra. Then, by Proposition 3.9, we have that for all $\psi \in$ T, $v(\psi) = \top$ but $v(\phi) < \top$. Hence T $\nvDash_{\mathbf{IMTL}}$ ϕ, where $_{\mathbf{IMTL}}$ is the set of complete **IMTL** frames. □

4. Concluding remark

Here we investigated just algebraic Routley-Meyer-style semantics for some **IMTL**. Note that Yang provided not only algebraic but also *set-theoretic* Kripke--style semantics for fuzzy logics in Yang (2015a; 2016c, 2017a, 2017b). This gives rise to a question to consider set-theoretic Routley-Meyer-style semantics for

IMTL. We leave this for an another work.

References

Ciabattoni, A., Esteva, F., and Godo, L. (2002) “T-norm-based logics with n-contraction”, *Special Issue on SOFSEM2002 of Neural Network World*, 12, pp. 453-460.

Cintula, P. (2006) “Weakly Implicative (Fuzzy) Logics I: Basic properties”, *Archive for Mathematical Logic*, 45, pp. 673-704.

Diaconescu, D. and Georgescu, G. (2007) “On the forcing semantics for monoidal t-norm based logic”, *Journal of Universal Computer Science*, 13, pp. 1550-1572.

Galatos, N., Jipsen, P., Kowalski, T., and Ono, H. (2007), *Residuated lattices: an algebraic glimpse at substructural logics*, Amsterdam, Elsevier.

Hájek, P. (1998), *Metamathematics of Fuzzy Logic*, Amsterdam, Kluwer.

Jenei, S and Montagna, F. (2002), “A Proof of Standard completeness for Esteva and Godo's Logic **MTL**”, *Studia Logica*, 70, pp. 183-192.

Kripke, S. (1963). “Semantic analysis of modal logic I: normal modal propositional calculi”, *Zeitschrift für mathematische Logik und Grundlagen der Mathematik* 9, pp. 67-96.

Kripke, S. (1965a) “Semantic analysis of intuitionistic logic I”, in J. Crossley and M. Dummett (eds.) *Formal systems and Recursive Functions*, Amsterdam: North-Holland Publ Co, pp. 92-129.

Kripke, S. (1965b) “Semantic analysis of modal logic II”, in J. Addison, L. Henkin, and A. Tarski (eds.) *The theory of models*, Amsterdam: North-Holland Publ Co, pp. 206-220.

Montagna, F. and Ono, H. (2002) “Kripke semantics, undecidability and standard completeness for Esteva and Godo's Logic IMTL∀”, *Studia Logica*, 71, pp. 227-245.

Montagna, F. and Sacchetti, L. (2003) “Kripke-style semantics for many-valued logics”, *Mathematical Logic Quaterly*, 49, pp. 629-641.

Montagna, F. and Sacchetti, L. (2004) “Corrigendum to “Kripke-style semantics for many-valued logics”, *Mathematical Logic Quaterly*, 50, pp. 104-107.

Urquhart, A. (1986) “Many-valued logic”. in D. Gabbay and F. Guenthner (eds.) *Handbook of Philosophical Logic*, vol. 3, Dordrecht: Reidel Publ Co, pp. 71-116.

Wang, S. (2012) “Uninorm logic with the *n*-potency axiom”, *Fuzzy Sets and Systems*, 205, pp. 116-126.

Wang, S. (2013) Involutive uninorm logic with the *n*-potency axiom“, *Fuzzy Sets and Systems*, 218, pp. 1-23.

Yang, E. (2014a) “Algebraic Kripke-style semantics for weakening-free fuzzy logics”, *Korean Journal of Logic*, 17 (1), pp. 181-195.

Yang, E. (2014b) “Algebraic Kripke-style semantics for relevance logics”, *Journal of Philosophical Logic*, 43, pp. 803-826.

Yang, E. (2014c) “Algebraic Kripke-style semantics for three-valued paraconsistent logic”, *Korean Journal of Logic*, 17(3), pp. 441-460.

Yang, E. (2016a) “Algebraic Kripke-style semantics for substructural fuzzy logics”, *Korean Journal of Logic* 19, pp. 295-322.

Yang, E. (2016b) “Weakly associative fuzzy logics”, *Korean*

Journal of Logic 19, pp. 437-461.

Yang, E. (2016c) “Basic substructural core fuzzy logics and their extensions: Mianorm-based logics”, *Fuzzy Sets and Systems* 276, pp. 43-58.

Yang, E. (2017a) “Set-theoretic Kripke-style semantics for involutive monoidal t-norm (based) logics”, *Philosophical Ideas*, 55(s), pp. 79-88.

Yang, E. (2017b) “A non-associative generalization of continuous t-norm-based logics”, *Journal of Intelligent & Fuzzy Systems* 33, pp. 3743-3752.

Yang, E. (2018) “Algebraic Routley-Meyer-style semantics for the fuzzy logic **MTL**”, *Korean Journal of Logic* 21(3), pp. 353-371.

Yang, E. (2019) “Mianorm-based logics with n-contraction and n-mingle axioms”, *Journal of Intelligent & Fuzzy Systems* 37, pp. 7895-7907.

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퍼지 논리 IMTL을 위한 대수적 루트리-마이어형 의미론

양 은 석

대수적 루트리-마이어형 의미론이라고 불리는 루트리-마이어형 의미론이 퍼지 논리 체계 **MTL**을 위하여 연구되었다. 우리는 이 연구를 누승적 체계들로 확장한다. 이의 한 예로 이 글에서 우리는 퍼지 논리 체계 **IMTL**을 위한 대수적 루트리-마이어형 의미론론을 연구한다. 먼저 퍼지 논리 체계 **IMTL**과 대수적 의미론을 소개한다. 다음으로 이 체계를 위한 대수적 루트리-마이어형 의미론을 제공한 후, 이를 대수적 의미론과 연관 짓는다.

주요어: (대수적) 루트리-마이어형 의미론, 크립키형 의미론, 퍼지 논리, 대수적 의미론, 준구조 논리