

:

()

1. 가

1931 가 ,
가 .

‘ ,
’(Reverse Mathematics Program) “

” .1)

‘ ’ 1974 (Harvey Friedman)

「2 가 」

(Friedman 1975)

1) “ ”(Reverse Mathematics Program)
Stephen G. Simpson (Simpson 1985, Simpson 1987, Simpson 1988)
‘ , ‘ , 1988
“ ”(Journal of Symbolic Logic, Vol. 53,
No. 2, 337-384) Roman Murawski (Murawski
1993, Murawski 1994)

가 ,
 '2 (subsystems of second
 order arithmetic) 가 .
 2
 (Hilbert-Bernays 1934, 1939)
 , 2 가 ,
 가 .2)
 가, 2
 가 ,
 ' ,
 1982 (Stephen G. Simpson)
 ' (REVERSE MATHEMATICS)
 ' (ordinary mathematics)
 (set-theoretic mathematics) ,
 (number theory),
 (geometry), (calculus) ' ,
 '(non-set-theoretic mathematics) .
 , (functional analysis),

2) Friedman 1975, 235 . Simpson(Simpson 1985, 462) 2
 ' (coding) 2
 '(conservative extensions) 가 .

, (general topology) .
 2 ,
 2

()
 ‘ (set existence axiom)

“ 가 가?” .
 2

:

, 가 가 가
 , 가 “ ” 가
 .)
 “

”(Simpson 1988, 355) 가

. , 1985 (APA)
 (ASL) 가 「

」(Simpson 1988)

3) Simpson 1985, 467 : “Very often, if a theorem of ordinary mathematics is proved from the weakest possible set existence axioms, it will be possible to “reverse” the theorem by proving that it is equivalent to those axioms over a weak base theory. This phenomena is known as REVERSE MATHEMATICS.”

‘reverse mathematics’

‘forward mathematics’

(Simpson 1988, 356)

가

. , 가
(「 」(Hilbert 1925))

“
”(Simpson 1988, 349)

(Solomon Feferman)

(relativization) “
”(Feferman 1993, 159) ,

(Roman Murawski) “
”(Murawski 1993, 181) 가

가

(finitism)가

‘ , ‘

(William Tait)가

(Primitive Recursive Arithmetic, PRA)

.4)

2.

, 1 (first order arithmetic)

Peano Arithmetic(PA)

(Primitive Recursive Arithmetic, PRA) 1

, 2 (second order arithmetic)

.5)

1 L₀ (type) 0 , 1 x, y,

z, . . . , 0, (successor symbol) ‘ ,

4) Ulrich Majer “*Dfferent forms of finitism*”(Majer 1993)

Weyl

“ ”

PRA

(Majer 1993, 189)

5)

Feferman 1988 Fefermann 1993

. Feferman PRA가 PA

(primitive recursive function) $+$, \cdot , \dots
 $f_0, f_1, \dots \quad \cdot L_0$

(term) $t_1, t_2, \dots \quad 0, \quad ' \quad fi$
 $\cdot L_0 \quad$ (atomic formula)

(equation) $t_1 = t_2 \quad \cdot L_0 \quad$ (formula)

$\neg, \quad , \quad ,$

(quantification) \cdot

$L_0 \quad \varphi \uparrow$ $, \quad \varphi \quad ' \quad$
 $'$ (quantifier-free) $, \quad \varphi \in QF \quad \cdot L_0$
 n

$, \quad QF \quad ,$

$\Sigma_n^0 \quad \cdot \quad \psi \in QF \quad \varphi$

$= (\exists y)(\forall x)(\exists z) \psi \quad , \quad \varphi \in \Sigma_3^0 \quad \cdot$

$L_0 \quad n$

$, \quad QF \quad ,$

$\Pi_n^0 \quad \cdot \quad \psi \in QF$

$\varphi = (\forall x)(\exists y) \psi \quad , \quad \varphi \in \Pi_2^0 \quad \cdot$

(arithmetic) $1 \quad ,$
 $: \quad x' \neq 0 \quad x' = y' \quad x$

$= y \quad x + 0 = 0 \quad x + y' = (x + y)' \quad x \cdot 0 = 0 \quad x \cdot y' = x \cdot y + x,$
 f

(Induction Axiom Scheme)

IA. $\varphi(0) \quad x(\varphi(x) \rightarrow \varphi(x')) \rightarrow x \varphi(x)$

L_0 $\varphi(x)$ 1
 (first order arithmetic) Peano Arithmetic(PA)

. 1 IA ,
 $\Sigma_0^0, \Sigma_1^0, \Sigma_2^0$ $\varphi(x)$ ' ' ,
 . 1
 $I \Sigma_0, I \Sigma_1, I \Sigma_2$.
 PA PRA L_0

QF

(Induction Rule)

IR. $\frac{\varphi(0), \varphi(x) \rightarrow \varphi(x')}{\varphi(x)}$

$\varphi \in QF$.
 2 L_1 1 L_0 1 ,
 (set variable) X, Y, Z, . . .

. L_1 , t

X t X .

L_1 X, Y

X Y .

L_1 QF, Σ_n^0, Π_n^0

L_0 . L_1

' (arithmetical)
 $A_{arith} \cup_n \Pi_n^0 \quad \Pi^0$
 $\cdot L_1$
 $\cdot A_{arith} \quad \Sigma_n^1$
 $\phi \in A_{arith} \quad \varphi = (\exists Y)(\forall X)$
 $(\forall Z) \psi \quad , \quad \varphi \in \Sigma_3^1 \quad \cdot \quad L_1$
 $\cdot \quad \Sigma_n^1 \quad , \quad A_{arith}$
 $\cdot \quad \Pi_n^1 \quad \cdot$
 $\phi \in A_{arith} \quad \varphi = (\forall X)(\exists Y) \psi \quad , \quad \varphi \in \Pi_2^1 \quad \cdot$
 $2 \quad \Sigma_2^1 \quad (\Pi^1 - CA_0) \quad 1$
 L_1
 $X \uparrow \quad L_1 \quad \varphi$
 (Comprehension Axiom) :

CA. $\exists x(x \in X \leftrightarrow \varphi(x)).$

2
 가 $RC A_0$
 \cdot
 $L_1 \quad \Sigma_1^0$
 $\Sigma_1^0 - IA, \quad CA$

(Recursive Comprehension Axiom)

$$\begin{array}{ccc} \text{R} & & \text{C} & & \text{A} \\ (\forall x)(\varphi(x) \leftrightarrow \psi(x)) \rightarrow (\exists X)(\forall x)(x \in X \leftrightarrow \varphi(x)), \varphi \in \Sigma_1^0, \psi \in \Pi_1^0 \end{array}$$

. 2

$RC A_0$

.

WKL_0 2

$RC A_0$

Weak König Lemma, “ 2 가

”(any binary branching infinite tree must contain some infinite path)

$$WKL. \quad (\exists X)(\text{BinTree}(X) \wedge \text{Inf}(X) \rightarrow (\exists Y)(\text{Path}(Y, X)))$$

.

T_1

T_2

Φ

(conservative)

:

T_1 is conservative over T_2 for Φ iff
 $\phi \in \Phi$ and $T_1 \vdash \phi$ implies $T_2 \vdash \phi$.

,

가

.

If T_2 is consistent, then T_1 is consistent.

3.

“ ”
 ‘ ’(formalized
 Hilbert's Program) .(Simpson 1988, 350~352)

가

,

.6)

가

“ ”(Simpson 1988, 352)

,

(Tait 1981)

가 PRA

2

, Z_2

6)

Alejandro
 Garciadiego(Garciadiego 1990), Marcus Giaquinto(Giaquinto 1983), Volker
 Peckhaus(Peckhaus 1994)

가 , Z_2 가 II_1^0 PRA 가 , Z_2 가 II_1^0 PRA 가 , Z_2 가 II_1^0 PRA : Z_2 가 II_1^0 PRA 7) Z_2 , PA II_1^0 , 1 PA Z_2 Z_2 , ‘ , ’ 2 WKL_0 (Wilfried Sieg), (Charles Parsons), (G. Minc), (Leo Harrington) 가 .

7) Simpson 1988, 352 : “ Z_2 is conservative over PRA with respect to II_1^0 sentences.”

WKL_0

8) :

(Friedman-Simpson) : RCA_0

WKL_0

Heine-Borel Theorem : Every covering of $[0, 1]$ by a countable sequence of open intervals has a finite subcoverings.

Every continuous function on $[0, 1]$ is uniformly continuous.

Every continuous function on $[0, 1]$ is bounded.

Every continuous function on $[0, 1]$ has a supremum.

The local existence theorem for solutions of ordinary differential equations.

Every countable commutative ring has a prime ideal.

Every countable formally real field can be ordered.

Every countable formally real field has a real closure.

Gödel's completeness theorem for predicate calculus.

WKL_0

2

가 가

9)

1977

(L. Kirby)

(J. Paris)

8)

S. Simpson

Subsystems of Second Order Arithmetic (Simpson 199?)

Simpson 1985, 468~469

9) WKL_0

“Bolzano-Weierstrass

Theorem: Every bounded sequence of real numbers has a convergent subsequence”

가 . Bolzano-Weierstrass Theorem WKL_0 Arithmetical Comprehension

Axiom

ACA_0

(Kirby-Paris 1977)가 “ WKL_0 가 Π_2^0 PRA
 ”10)

WKL_0 가 Π_1^0
 PRA , WKL_0
 ‘ ,
 . (Sieg 1985)

PRA
 :
 WKL_0
 .11)

“
 가 ”12)

4.

‘ (evolutionary program) ,
 ,

10) Simpson 1988, 353 : “ WKL_0 is conservative over PRA with respect to Π_2^0 sentences.”
 11) Simpson 1988, 354 . : “Any mathematical theorem which can be proved in WKL_0 is finitistically reducible in the sense of Hilber's program.”
 12) Simpson 1988, 349 : “the feasibility of a significant *partial* realization of Hilbert's program”.

1927

“ (Hilbert 1927) (L. E. J. Brouwer)

, :

가 가

, ,

.13)

(Volker Peckhaus) “

”(Peckhaus 1994) , 가 1900

‘ ,

.14) 1900 1904

가 “ ,

”(Peckhaus 1994, 95)

13) Hilbert 1927, 475 : “The formula game that Brouwer so deprecates has, besides its mathematical value, an important general philosophical significance. For this formula game is carried out according to certain definite rules, in which the technique of our thinking is expressed. . . . The fundamental idea of my proof theory is none other than to describe the activity of our understanding, to make a protocol of the rules according to which our thinking actually proceeds.”

14) “Hilbert , ‘ , ’ (Hilbert's axiomatization of geometry served as a model for the *mathematical* part of critical mathematics, which was to be supplemented, so to speak, with a ‘philosophical fundament’ provided by subsequent *philosophical* efforts. p. 100, in Peckhaus 1994)

1903
 , 가
 , 가
 - .15)
 “ ”
 가 가 1900
 “ ”(Hilbert 1900)
 “ 가
 ” , 가 “
 ” “
 ”
 가 , “
 가 ”16)

15) A. Garciadiego(Garciadiego 1990, 248) : “by 1891, some years before Hilbert had any knowledge either of the set theoretic paradoxes or the resulting crises concerning the foundations of mathematics, he already had in mind some of the elements that would later constitute his formalist program.”

16) Hilbert 1900, 7 : “in mathematics there is no *ignorabimus*.”
 가 30 (Hilbert 1930)
 “ignorabimus [We will never know]” , “
 ”(We must know, we will know)

5.

가

PRA

”

.17)

(Ulrich Majer)

”(Majer 1993)

가

18)

가

:

D)

가

PRA

II)

가

17) Simpson 1988, 352 : “Hilbert's finitism is captured by the formal system *PRA* of primitive recursive arithmetic (also know as Skolem arithmetic). . . . I am going to accept Tait's identification of finitism with *PRA*.”

18) Majer 1993, 185 : “I do not agree with the generally accepted view that Hilbert's finitism is identical or equivalent with primitive recursive arithmetic [*p.r.a.*] as Tait proposed.”

, 가
 , 가
 III) “ ”
 , (a universal principle
 of epistemology)
 가 (mathematical)

“ ”
 ”가 “ ” , 가
 “ ”(the finitist frame of mind)
 “ ”(a concern for concrete content)
 , “ ”
 :

, 가 , , . 19)

, 가

19) Hilbert 1925, 376 .

가 (finitary number theory)
 , (real proposition)
 (finitism)
 , “
 ”
 가
 ‘ ,
 ,
 (Marcus Giaquinto)가
 20 “ ”(a return to
 empiricism)
 가 . “ ”(Giaquinto 1983)
 가
 “ 가 2 (,
) ”20)
 ,
 , 가
 ‘ , ‘ ,가 “
 ”(empiricism), , “ 가
 ”(the only facts are observable facts)

20) Giaquinto 1983, 127 : “If we find it difficult to understand why, before Gödel’s results, Hilbert seemed to entertain no doubts about the possibility of achieving his programmatic objectives, it may be because we find ourselves in a philosophical climate generally hostile to the empiricism (specifically, positivism) which flourished before the Second World War.”

'(ideal propositions)
'(real propositions)
(reductionistic program)

6.

가
가
가

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